QUANTUM SUPPRESSION OF ION ACOUSTIC INSTABILITY IN DENSE PLASMA

Tonuj Deka*[1], Niranjan Gogoi[2], and Nilakshi Das[2]

[1]*Department of Physics, Tezpur College, Tezpur- 784001, Assam, India*
[2]*Department of Physics, Tezpur University, Tezpur- 784028, Assam, India*

*Corresponding Author: tonuj.tc@gmail.com

(Received 25 September 2025; revised 22 December 2025; accepted 24 December 2025; published 1 August 2026)

Abstract: A 1-D quantum hydrodynamics (QHD) model has been used to investigate the stability criteria of the ion-acoustic wave in a dense electron-ion quantum plasma. The ion-acoustic instability caused by electron streaming is well understood in classical plasma; however, when quantum effects are considered, the stability criteria undergo significant modifications. In the present work, electrons are described by the quantum fluid equation with $H = \frac{\hbar\omega_{pe}}{2k_B T_{Fe}}$ representing the quantum diffraction of the electron wavepacket. In classical plasma, even a small value of electron streaming velocity is enough to trigger ion acoustic instability; however, in presence of parameter H, a much higher value of electron streaming velocity is required to excite the instability. In this work, it is observed that the electron streaming contributes to the excitation of the instability, which gets suppressed due to the quantum effects of the electron, whereas the streaming ion provides energy to the wave mode.

Keywords: Quantum Plasma; Quantum Hydrodynamics; Quantum Ion Acoustic Wave

PACS: 52.35.-g; 61.10.Jn; 52.27.Cm

1 Introduction

Quantum plasma is a system of charged particles showing both collective and quantum effects [1]. Although the plasma is considered to be classical, the quantum mechanical behaviour of the constituent particles in a plasma are observed only at very large densities and relatively low temperatures in such a scenario, where the inter particle distance becomes almost equal to the de-Broglie wavelength [1, 2]. In such circumstances, the many particle system can be well defined by the Fermi Dirac statistics instead of Maxwell-Boltzmann statistics, preferably used in laboratory and space plasmas. However, the ions are considered to be nondegenerate due to their heavier masses as compared to the electron masses. Recently, quantum plasma has drawn immense interest due to its relevance in describing quantum effects observed in various environments such as in dense astrophysical objects [3], in ultrasmall electronic devices [4] and in intense laser plasma experiments [5].

During the last few decades, a great deal of interest has grown in the study of linear and nonlinear properties of electrostatic [6–9] and electromagnetic waves [10–12] in the quantum plasma. Haas et al. [6] utilized a nonlinear Schrodinger-Poisson system to develop a quantum multistream model and analysed the dispersion relations for plasma instabilities. They concluded that the reduced quantum effects in the plasma significantly amplify the two-stream instability. Manfredi et al. [7] derived an effective Schrodinger-Poisson model by taking the moments of the Wigner equation and applied it to investigate linear wave propagation and two-stream instability for a system of degenerate electron gas. Anderson et al. [13] did similar work to investigate various effects in quantum plasma using the Wigner-Poisson system. They demonstrated the suppression of instabilities in one and two stream plasma by a Landau-like damping mechanism. The characteristics of linear and nonlinear ion acoustic waves (QIAW) in a one-dimensional quantum plasma were investigated by Haas et al. [8] using the quantum hydrodynamic model (QHD). Ghosh et al. [14] too had used one-dimensional QHD model to investigate the linear and nonlinear properties of electron plasma waves in a two-dimensional quantum plasma. The QHD model is basically analogous to the classical fluid model containing a quantum correction term called the Bohm potential. Its simplicity and numerical efficiency have led to its usage in numerous linear and nonlinear studies of quantum waves in quantum plasma [15].

The streaming instability in various quantum plasma systems has attracted significant interest from researchers due to several fundamental reasons applicable to both laboratory settings and compact astrophysical environments [16]. In this present work, we have studied the linear wave characteristics of the quantum ion acoustic wave with electron and ion streaming in a quantum plasma in the core of white dwarf star. The dispersion relation of the quantum ion acoustic wave has been derived using a one-dimensional quantum hydrodynamics model. The model consists of the continuity equations, momentum equations with the Bohm potential term and Poisson's equation. The effect of the electron to ion mass ratio and quantum effects on the wave are also investigated here. We have further investigated the instability triggered by electron streaming velocity in the quantum regime of ion acoustic wave. In section 2, we present the QHD model used for our quantum plasma system in details. Section 3 deals with the results of this study and discussions were presented for the plasma in the white dwarf star's core. In section 4 we present the study of the instability triggered by the streaming electrons in the white dwarf plasma environment. Finally, section 5 contains the conclusion.

2 Theoretical Model

In this work, we consider a 1-D quantum hydrodynamic model to describe the propagation of a quantum ion acoustic wave in a two-component quantum plasma. Here, the ion mass provides the inertia while the restoring force is provided by the Bohm potential of the electrons and the degeneracy pressure. At very high density, the de Broglie wavelength of electrons becomes of the order of inter-particle separation. The quantum diffraction effect arising due to this wave nature of electrons is introduced to the system via the Bohm potential term. Electron and ion streaming velocities u_{e0} and u_{i0}, respectively, are included in the respective equations. Such flow may be important in quantum plasma system such as white dwarf, caused by accretion flow. This 1-D QHD model consists of continuity and momentum equations for the electrons and ions together with Poisson's equation for the self-consistent potential as described below:

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{e}{m_e}\frac{\partial \varphi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial P_e}{\partial x} + \frac{\hbar^2}{2{m_e}^2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2 \sqrt{n_e}}{\partial x^2}\right) \tag{3}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \varphi}{\partial x} + \frac{\hbar^2}{2{m_i}^2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_i}}\frac{\partial^2 \sqrt{n_i}}{\partial x^2}\right) \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{e}{\epsilon_0}(n_e - n_i) \tag{5}$$

Here n_i (n_e), u_i (u_e), m_i (m_e), e ($-e$) are the number density, velocity, mass and charge of the ions (or electrons) respectively and, $\hbar$ and ϵ_0 are the reduced Planck's constant and dielectric constant respectively. Here, 1-D system in the electrostatic approximation with potential φ is being considered. The last term on the right hand side of the Eqs. (3) and (4) is the Bohm potential term, the term related to the quantum diffraction effects present in the system.

Here, the following normalization are being used for further calculations: $\bar{t} = \omega_{pi} t$, $\bar{x} = \omega_{pi} x / c_s$, $\overline{n_e} = n_e/n_0$, $\overline{n_i} = n_i/n_0$, $\overline{u_e} = u_e/c_s$, $\overline{u_i} = u_i/c_s$, and $\overline{\varphi} = e\varphi/2k_B T_{Fe}$. Here, $\omega_{pi} = \sqrt{\frac{n_0 e^2}{m_i \epsilon_0}}$ is the ion plasma frequency n_0 is the equilibrium number density, k_B is the Boltzmann constant, T_{Fe} is the Fermi temperature, and $c_s = \sqrt{\frac{2k_B T_{Fe}}{m_i}}$ is the ion acoustic velocity. Along with these, the nondimensional quantum parameter H ($H = \frac{\hbar \omega_{pe}}{2k_B T_{Fe}}$, where $\omega_{pe} = \sqrt{\frac{n_0 e^2}{m_e \epsilon_0}}$ is the electron plasma frequency) is used to describe the quantum diffraction of the electron wavepacket. Physically, this quantum parameter H signifies the ratio of the electron plasmon energy to electron Fermi energy and measures the strength of the quantum diffraction effects.

Using the normalization parameters and dropping the bars, Eqs. (3) and (4) becomes

$$\mu\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)=\frac{\partial\varphi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right) \tag{6}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial\varphi}{\partial x}+\frac{m_e}{m_i}\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_i}}\frac{\partial^2\sqrt{n_i}}{\partial x^2}\right) \tag{7}$$

The ion momentum equation contains a Bohm potential term. However, we can neglect the term in the Eq. (7) for the ions with the argument that the quantum diffraction effects for the ions are negligible due to their heavier masses. Using Eqs. (5),(6), and (7), the dispersion relation for linear waves in the two-component quantum plasma is obtained, which is given by

$$1-\frac{1}{\mu(\omega-ku_{e0})^2-k^2\left(1+\frac{k^2H^2}{4}\right)}-\frac{1}{(\omega-ku_{i0})^2}=0 \tag{8}$$

Here $\mu=\frac{m_e}{m_i}$ is the ratio of electron to ion masses. The quantum parameter H introduces the additional dispersive effect to the propagation of the acoustic mode. The wave is further affected due to the presence of ion-electron streaming.

3 Results & Discussions

To understand the properties of the quantum ion acoustic wave, the dispersion relation is interpreted on the basis of typical parameters for the core of white dwarf star: quantum parameter $H \sim 0.3$ [17], fermi temperature $T_{Fe} \sim 10^5$ eV [17], ion temperature $T_i \sim 10^3$ eV [18]. The typical ion and electron streaming velocity are $u_{i0} \sim 0.06-0.07$ and $u_{e0} \sim 18.38-31.82$ respectively (as estimated from [19]). White dwarf stars go through several cooling phases after the completion of thermonuclear reactions such that the cores of such stars may attain mass density of $\rho \sim 10^6-10^7 gm/cm^3$, and temperature $T \leq 10^8 K$. In such extreme conditions, the quantum effects become important for electrons, thus providing an ideal test bed for quantum ion acoustic wave. To understand the linear ion acoustic wave mode of the dense plasma in the white dwarf star, the dispersion relation is plotted for different quantum parameter *H*, electron and ion streaming velocities corresponding to the white dwarf environment. The dispersion relation given by Eq. (8) can be further simplified to

$$\omega(k)-ku_{i0}=k\left(\sqrt{\frac{4+k^2H^2-4\mu u_{e0}^2}{4+k^4H^2+4k^2(1-\mu u_{e0}^2)}}\right)=\omega'(k) \tag{9}$$

where both streaming electrons and ions play a significant role in the dispersion properties of the quantum ion acoustic wave. For small wavenumbers, $\omega \approx k$ suggests that a wave propagates in the system with the phase velocity, $v_p=\sqrt{\frac{4+k^2H^2-4\mu u_{e0}^2}{4+k^4H^2+4k^2(1-\mu u_{e0}^2)}}+u_{i0}$. This wave mode is analogous to the classical ion-acoustic wave mode and hence can be termed as quantum ion acoustic wave [8]. While ion streaming contributes via Doppler shifted frequency, electron streaming may excite streaming instability of the QIAW mode.

Fig. 1 shows the dispersion relation for the linear waves without the streaming electron and ion for three different values of quantum parameter *H*, namely *H* = 0, 0.5, 1.0. Similar to the classical mode, this quantum wave mode represents the electrons and ions oscillations at low frequency. Hence, at a smaller wavelength or at a larger wavenumber, the electrons and ions oscillate at the ion plasma frequency, ω_{pi}. Here, in Fig. 1, the curve corresponding to *H* = 0 represents the absence of any quantum effects due to high density in the white dwarf core, thus representing the classical ion acoustic wave. It is seen that with the increase of the parameter *H*, the wave reaches faster to the ω = 1 value. In absence of streaming, the results agree with those obtained by [8] in their study on quantum ion acoustic wave in quantum plasma. Here, the quantum ion acoustic wave describes the low-frequency density fluctuations near the ion plasma frequency $\omega_{pi}=3.30\times10^{14}$ Hz for the values of considered parameters. This is found to be significantly higher than the ion acoustic wave excited in laboratory plasma. In the limit u_{e0}, u_{i0}, $H\to0$, Eq. (9) reduces to the usual IAW dispersion relation. Putting the above values in Eq. (9), the typical frequency of the QIAW is $0.77\omega_{pi}$ which is nearly equal to 2.55×10^{14} Hz for Oxygen ions of mass

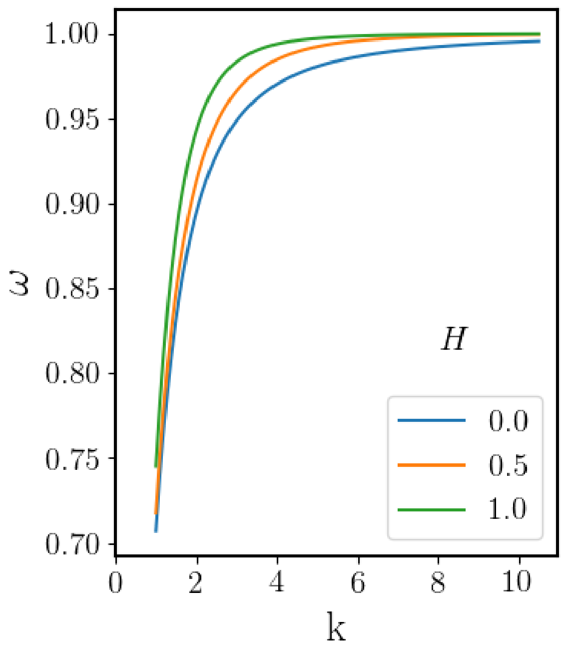

Figure 1: Dispersion relation for three different values of quantum parameter H, ($H \sim 0.0, 0.5, 1.0$) without any streaming electrons and ions in the white dwarf core plasma.

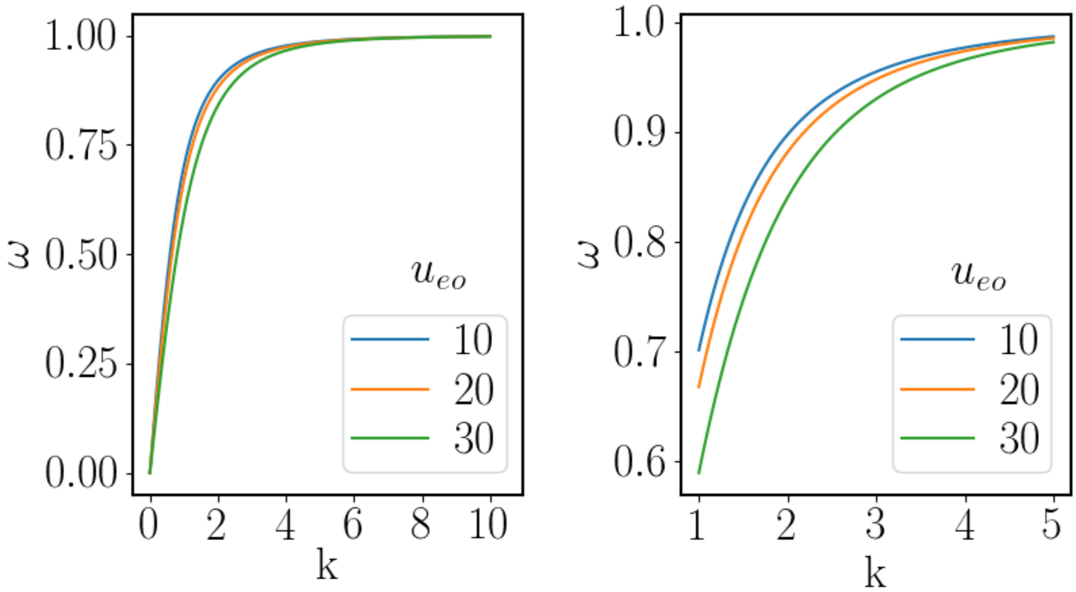

Figure 2: The image on the left panel shows dispersion relation with normalized electron streaming velocities $u_{eo} = 10, 20, 30$ and ion streaming velocity $u_{io} = 0$ for $H = 0.3$. The image on the right panel shows the zoomed image of the dispersive curve in the range $1 < \mathrm{k} < 5$.

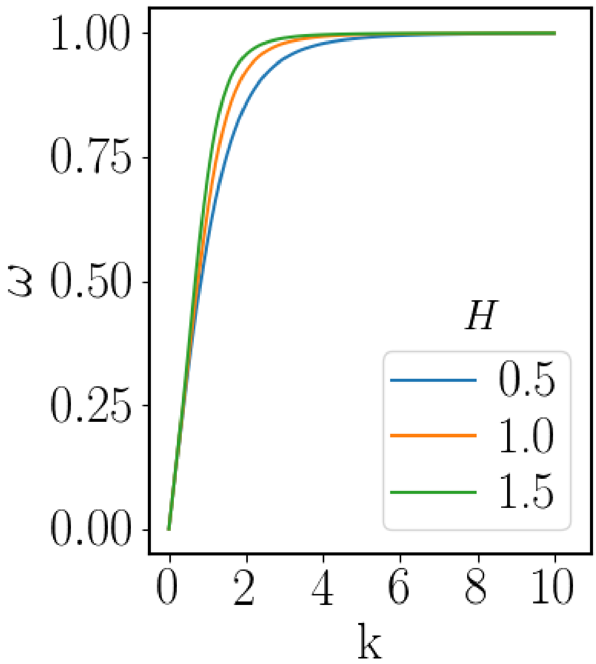

Figure 3: Dispersion relation with normalized electron streaming velocity $u_{e0} = 32$ and ion streaming velocity $u_{i0} = 0$ for $H = 0.5, 1.0, 1.5$.

2.66×10^{-26} kg and the number density considered is $10^{30} m^{-3}$. Hence, at a smaller wavelength or at a larger wavenumber, the electrons and ions oscillate at the ion plasma frequency, ω_{pi}.

We have seen that Eq. (8) and Eq. (9) contains both the electron and ion streaming velocities. The effect of the streaming electrons on the quantum ion acoustic wave mode is presented in Fig. 2. To verify this effect, we have considered three values of the normalized electron streaming velocity $u_{eo} = 10, 20, 30$, and neglected the ion streaming velocity ($u_{io} = 0$). Here, the dispersion relation is plotted for the parameter *H* = 0.3. It is observed in Fig. 2 that as the electron streaming velocity increases, the slope of the dispersion curve decreases, thereby suggesting a decrease of the wave velocity. This effect of the increase of the electron streaming velocity is clearly visible in the right-hand image of Fig. 2. Moreover, these streaming electrons have no effect on the influence of the parameter *H* on the wave mode. The influence of the increase of *H* on the quantum ion acoustic wave is the same even in the presence of the streaming electrons, as evident from Fig. 3. Fig. 3 shows the dispersion relation considering streaming electrons with the streaming velocity $u_{eo} = 32$ and neglecting the ions streaming for three different values of $H = 0.5, 1.0, 1.5$.

The effect of the streaming ions on the quantum acoustic wave mode is also investigated. Ignoring the electron streaming velocity, we have plotted the dispersion relation taking the ion streaming velocities $u_{io} = 0.0, 0.06, 0.07$ and the parameter $H = 0.3$ in Fig. 4. When the ion streaming velocity is considered, the wave no longer saturates and continues its propagation. As the wavenumber k increases, the wave frequency keeps on increasing. This can be considered to be due to the fact that as the velocity of the streaming ions increases, these ions provide energy to the wave propagation; as a result, the wave no longer saturates at the ion plasma frequency and keeps on propagating. However, at $u_{i0} = 0$ i.e. in the absence of the streaming ions, the wave saturates similarly to the normal ion acoustic wave.

The impact of the quantum parameter on the wave mode in the presence of the streaming ions is also investigated. Here, the dispersion relation of the wave is plotted in Fig. 5 for three different values of the parameter $H = 0.5, 1.0, 1.5$ with normalized ion streaming velocity $u_{io} = 0.06$ and negligible electron streaming velocity $u_{eo} = 0$. Fig. 5 shows the effect of the streaming ions i.e. the bending of the saturation curve, along with the effect of the increase of the parameter *H*. Similar to the case of no streaming, the wave here reaches faster to the $\omega = 1$ value with the increase of the parameter, even with the streaming ions.

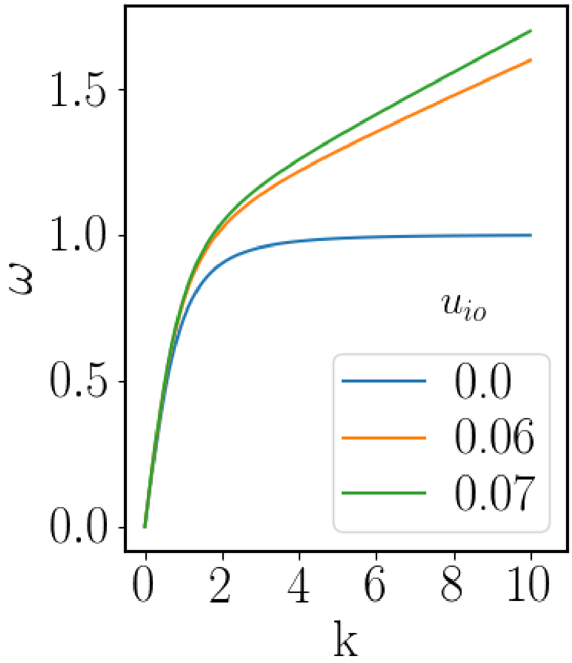

Figure 4: Dispersion relation with normalized ion streaming velocities $u_{io} = 0.0, 0.6, 0.7$ and electron streaming velocity $u_{eo} = 0$ for $H = 0.3$.

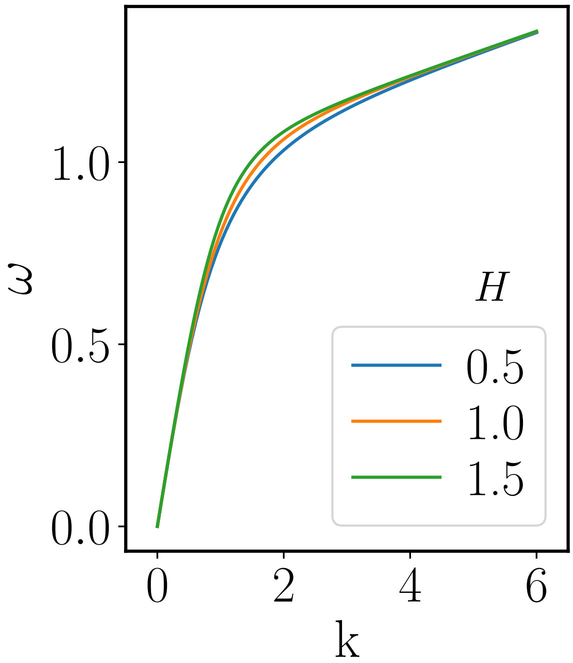

Figure 5: Dispersion relation showing the effect of the increase of the quantum parameter H ($H \sim 0.5, 1.0, 1.5$) on the wave mode in the presence of the streaming ions with normalized streaming velocity u_{io}= 0.06 and electron streaming velocity $u_{e0} = 0$.

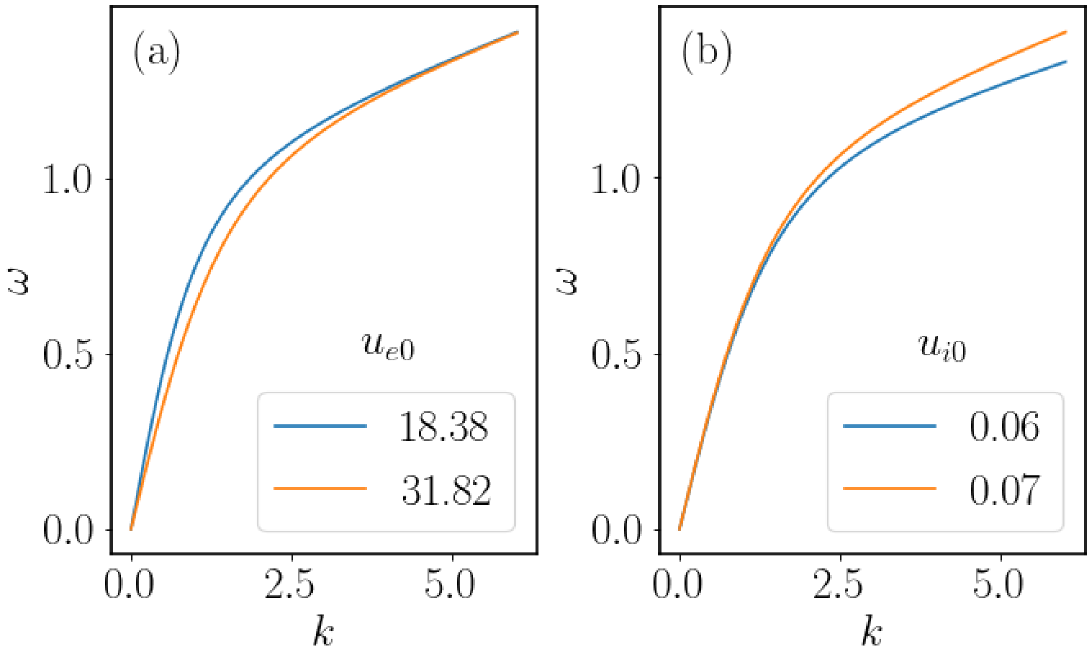

Figure 6: Dispersion relation with normalized electron and ion streaming velocities for (a) $u_{io} = 0.07$ and $u_{eo} = 18.38, 31, 82$, (b) $u_{eo} = 31.5$ and $u_{io} = 0.06, 0.07$.

The combined effect of electron and ion streaming velocities on the quantum ion acoustic wave mode can be seen from the plots of Figs. 6 (a) and 6 (b). The ion streaming velocity mainly contributes towards adding a constant velocity to the phase velocity of the wave mode. On the other hand, small values of u_{e0} have only a negligible effect on the dispersion curve. The electron streaming velocity starts affecting the wave significantly only in the presence of streaming ions and the effect is mainly revealed in the long wavelength regime (lower value of wavenumber). The role of electron streaming is further investigated in the next section in the context of streaming-induced instability in the presence of quantum effects. In Fig. 6 (a) the dispersion relation is shown for two values of the electron streaming (u_{e0} = 18.38 and 31.82), considering constant ion streaming velocity ($u_{i0} = 0.07$). It has been observed that at sufficiently high electron streaming velocities, a change in the velocity can decrease the slope of the curve, hereby suggesting that with the increase of the streaming velocity of the electrons, the phase velocity of QIAW decreases. Fig. 6 (b) is plotted for two values of ion streaming velocities $u_{io} = 0.06$ and 0.07 and electron streaming velocity $u_{eo} = 31.5$. Here, it is observed that as ion streaming velocities increase, the wave no longer saturates. In both the situations (Fig. 6 (a) and 6 (b)) the common phenomenon observed is that in the presence of both electron and ion streaming velocities, the wave no longer saturates, and as wavenumber k increases, the wave frequency keeps on increasing.

4 Stability Analysis

An analysis of the dispersion relation Eq. (9) gives important insight into the role of electron streaming on the instability of quantum ion acoustic wave mode. It has been observed that the quantum parameter H has a stabilizing effect, whereas electron streaming may trigger the instability. A straightforward analysis of Eq. (9) gives the boundary of the unstable regime as $1 + \frac{1}{4}k^2H^2 < \mu u_{e0}^2 < 1 + \frac{1}{k^2} + \frac{1}{4}k^2H^2$. Fig. 7 depicts the boundaries of the unstable regime in the $(\mu u_{e0}^2 - k)$ plane for various values of parameter H. For a higher value of H, a higher value of electron streaming velocity u_{e0} is required to trigger the instability. The above boundary condition for the unstable region may be stated in terms of the quantum parameter H as $4(\mu u_{e0}^2 - 1) - \frac{4}{k^2} < k^2H^2 < 4(\mu u_{e0}^2 - 1)$. At $k = 1$, the condition may be written as $4(\mu u_{e0}^2 - 2) < H^2 < 4(\mu u_{e0}^2 - 1)$. This implies a cut-off value of μu_{e0}^2 as 2 to trigger the instability. When μu_{e0}^2 attains the cutoff value, the boundary condition for the unstable regime at $k = 1$ may be stated as $0 < H < 2$. In the core of white dwarf stars with $n_0 = 10^{30}/cm^3$ and $T_{Fe} \sim 10^5$ eV, the plasmon energy becomes $\hbar\omega_{pe} \sim 3.5 \times 10^4$ eV so that $H \approx 0.3$, thus satisfying the condition.

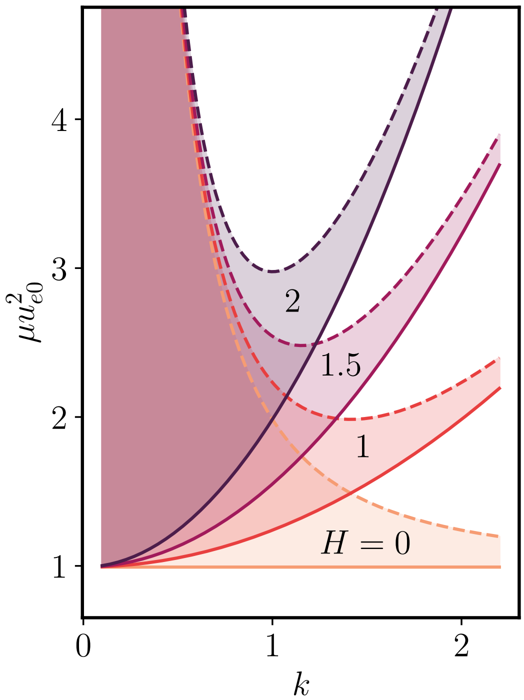

Figure 7: Stability curve of quantum ion acoustic wave in white dwarf plasma environment. The shaded parts denote the unstable region in the $(\mu u_{e0}^2 - k)$ parameter space for various values of H.

5 Conclusions

A 1-D quantum hydrodynamic model is used in this work to study the role of electron and ion streaming on the characteristics of the ion acoustic wave's dispersion relation in the dense plasma, where quantum effects become dominant for the lighter species of plasma. The results are tested in the quantum plasma that may exist in the core of white dwarf star. It has been observed that in the presence of the streaming ions, the wave no longer saturates at the ion plasma frequency. The wave frequency is found to increase with the increase of the wave number. It appears as if the streaming ions are providing energy to the wave, as a result, it no longer saturates.

A stability analysis is carried out for the streaming-induced instability of the quantum ion acoustic wave. The lower and upper boundaries of the reduced electron streaming velocity, viz. μu_{e0}^2 is derived in terms of the quantum parameter H. While the electron streaming triggers the instability, the quantum parameter H has a stabilizing effect. The higher the value of H, the higher μu_{e0}^2 is required to trigger the instability. The electron streaming induced instability may be triggered in quantum plasma when energy associated with the collective oscillations of electrons is within the limit of Fermi energy for $k \sim 1$. For higher value of wave vector k i.e. low wavelength regime, the range of H is reduced to excite the instability. The results may give an important clue to the investigation of ion acoustic mode in the interaction of an electron beam with dense laser plasma or astrophysical quantum plasma.

Acknowledgements

Niranjan Gogoi gratefully acknowledges the University Grants Commission (UGC), New Delhi-110002, for the financial support provided during the research under the Junior Research Fellowship (JRF) scheme, Ref no.: 221610069839.

References

[1] F. Haas, *Quantum Plasmas: An Hydrodynamic Approach* (Springer Science & Business Media, New York, 2011).

[2] F. Haas, Braz. J. Phys. **41**, 349 (2011).

[3] M. Opher et al., Phys. Plasmas **8**, 2454 (2001).

[4] P. A. Markowich, C. A. Ringhofer, C. Schmeiser, *Semiconductor Equations* (Springer Science & Business Media, New York, 2011)

[5] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).

[6] F. Haas, G. Manfredi and M. Feix, Phys. Rev. E **62**, 2763 (2000).

[7] G. Manfredi and F. Haas, Phys. Rev. B **64**, 075316 (2001).

[8] F. Haas *et al.*, Phys. Plasmas **10**, 3858 (2003).

[9] P. K. Shukla and B. Eliasson, Phys. Rev. Lett. **96**, 245001 (2006).

[10] P. K. Shukla, Phys. Lett. A **369**, 312 (2007).

[11] P. K. Shukla and L. Stenflo, Phys. Plasmas **13**, 044505 (2006).

[12] C. Li *et al.*, Phys. Plasmas **21**, 072114 (2014).

[13] D. Anderson *et al.*, Phys. Rev. E **65**, 046417 (2002).

[14] B. Ghosh, S. Chandra and S. N. Paul, Phys. Plasmas **18**, 012106 (2011).

[15] S. A. Khan and M. Bonitz, in *Complex Plasmas: Scientific Challenges and Technological Opportunities* (Springer, Cham, 2014).

[16] F. Cap et al., *Waves and Instabilities in Plasmas* (Springer, Vienna, 1994).

[17] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. **83**, 885 (2011).

[18] V. E. Fortov, Phys.-Usp. **52**, 615 (2009).

[19] M. Rosenberg, J. Plasma Phys. **67**, 235 (2002).