EXTERNAL CHARGED DEBRIS INDUCED NONLINEAR STRUCTURES IN A FLOWING PLASMA

Bikramjit Joardar[*1] and Madhurjya P. Bora[1]

[1]*Department of Physics, Gauhati University, Guwahati 781014, Assam, India*

[*]Corresponding Author: bikramjitjoardar1@gmail.com

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Abstract: In this work, we investigate the nonlinear structures triggered by periodically fluctuating external charged debris in a flowing plasma. Using the forced Korteweg-de Vries (fKdV) model together with Particle-in-Cell (PIC) simulations, we demonstrate that the periodic fluctuation suppresses the pinned solitons excited by the negatively charged debris whereas the dispersive shock waves (DSWs) excited by the positively charged debris are least affected. These findings highlight the role of debris charge fluctuation in plasma dynamics and its implications for debris-induced phenomena in the low Earth orbit (LEO) plasma.

Keywords: Debris Charge Fluctuation; Solitons; Dispersive Shock Waves

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1 Introduction

The subject of nonlinear structures driven by moving charged debris in a space plasma has received considerable attention not only due to their intricate nature but also due to the risks posed by such foreign entities to the numerous satellites orbiting the Earth. The topic of interaction of such objects with low Earth plasmas has gained prominence since the early space missions of the 1960s, which started studying the interaction between satellites/spacecrafts and space plasma environments [1–3]. It is now well established that such space debris can excite pinned, precursor and wake solitons as well as collisionless or dispersive shocks, depending on the velocity and polarity of such debris [4–6]. Experimental investigations have also confirmed the excitation of pinned solitons in the dust-acoustic regime in plasmas [7]. Space debris are also known to excite envelope compressional Alfvénic solitons as well as electromagnetic pinned solitons in magnetized plasmas [8]. Another study has also explored the properties of such solitons like amplitude, width and production frequency relating them with the size, velocity and position of the debris [9].

On the theoretical side, solitons are described by the Korteweg-de Vries (KdV) equation, arising out of the balance between nonlinearity and dispersion [10]. Envelope solitons are another type of nonlinear structure that can propagate while maintaining their shape because of their slowly varying envelope which modulates the comparatively faster carrier wave. Such envelope solitons can also result from a balance between nonlinearity and dispersion and is one of the solutions of the nonlinear Schrödinger equation (NLSE) [11]. The NLSE is also known to exhibit shock wave solutions, which appear in cases where the nonlinearity dominates over dispersion causing a steepening of the wavefront [12]. Such shocks can be either monotonic or oscillatory in nature and unlike solitons, they can evolve with time [13, 14]. In the context of the space debris problem, the fluid equations of the plasma can be reduced into a forced KdV (fKdV) equation, where the forcing term accounts for the charged debris. The fKdV equation can also exhibit solitons and dispersive shock wave solutions. It is worth noting that the soliton solution of the fKdV equation isn't actually a true soliton in the sense that a true soliton maintains its shape over time, unlike the fKdV solitons, which are unstable and originate from a non-integrable system [15].

In this work, we explore the effect of a periodic charge fluctuation of the debris on such nonlinear structures, for both positively and negatively charged debris. Our work is primarily focussed on the low Earth orbit (LEO) plasma where the context of plasma-debris interaction is most relevant. There could be many reasons for the variation of the debris charge including periodic photo-emissions due to eclipse transitions, debris in density gradients in the ambient plasma, etc. Among these, the most prominent mechanism is the debris charge fluctuation due to interactions with plasma waves, as it is a natural process. Debris in a plasma will inherently excite an ion-acoustic wave (IAW), which induces periodic oscillations in the ion and electron densities. This leads to a variation in the ion and electron currents reaching the debris surface. As a result, the equilibrium floating potential is disturbed and the debris charge evolves dynamically to restore the equilibrium, leading to periodic fluctuations in the debris charge [6, 16].

The manuscript is structured as follows. The governing fluid equations of the plasma-debris interaction are introduced in Section. 2. Section. 3 presents the forced Korteweg-de Vries (fKdV) analysis, and Section. 4 discusses the corresponding Particle-in-Cell (PIC) simulation results. Section. 5 concludes the paper.

2 The Plasma Model

The plasma model under consideration is 1-D, collisionless, and unmagnetized, with thermal ions and Boltzmannian electrons in the ion-acoustic regime. The external charged debris are embedded into the plasma. The relevant equations for this plasma are the ion continuity and momentum equation with the system being closed by the Poisson equation. The equations are as follows

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{1}{m_i n_i}\frac{\partial p_i}{\partial x} - \frac{e}{m_i}\frac{\partial \phi}{\partial x}, \tag{2}$$

$$n_e = n_0 \exp\left(\frac{e\phi}{T_e}\right), \tag{3}$$

$$\epsilon_0 \frac{\partial^2 \phi}{\partial x^2} = e(n_e - n_i) + \rho_{ext}(t, x - v_d t), \tag{4}$$

where n_i and n_e denote the respective number densities of ions and electrons, and v_i, ϕ, and p_i represent the ion fluid velocity, electrostatic plasma potential, and ion pressure respectively. The quantities e, m_i, n_0, and T_e correspond to the electronic charge, ion mass, equilibrium plasma density, and electron temperature (in energy units) respectively. The term ρ_{ext} represents the time-dependent external charged debris moving with velocity v_d with respect to the background plasma. Depending on the charge of the debris, ρ_{ext} can be $\gtrless 0$. The ion pressure is assumed to be polytropic

$$p_i \propto n_i^{\gamma}, \tag{5}$$

with γ being the ratio of the specific heats and is equal to 2 for our current 1-D model. Upon normalization, the above model becomes

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{6}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\gamma\sigma n_i^{\gamma-2}\frac{\partial n_i}{\partial x} - \frac{\partial \phi}{\partial x}, \tag{7}$$

$$n_e = e^{\phi}, \tag{8}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + \rho_{ext}(t, x - v_d t), \tag{9}$$

where the ion and electron densities have been normalized by the equilibrium value n_0, velocities by the ion-sound speed $c_s = \sqrt{T_e/m_i}$, plasma potential ϕ by (T_e/e). Time is normalized by the inverse ion plasma frequency, ω_{pi} while length has been normalized by the electron Debye length, λ_D.The parameter σ denotes the ion-to-electron temperature ratio. The debris term, ρ_{ext} has also been normalized accordingly.

The debris charge density ρ_{ext} can be further expanded as

$$\rho_{ext}(x, t) = \rho(x) f(t), \tag{10}$$

where $\rho(x)$ denotes the spatial distribution of the debris and $f(t)$ represents the time-variation of the debris charge. In this work, the debris charge fluctuation is controlled externally as modeling of a self-consistent periodic fluctuation would require calculation of the collision cross-sections and subsequent ion and electron currents to the

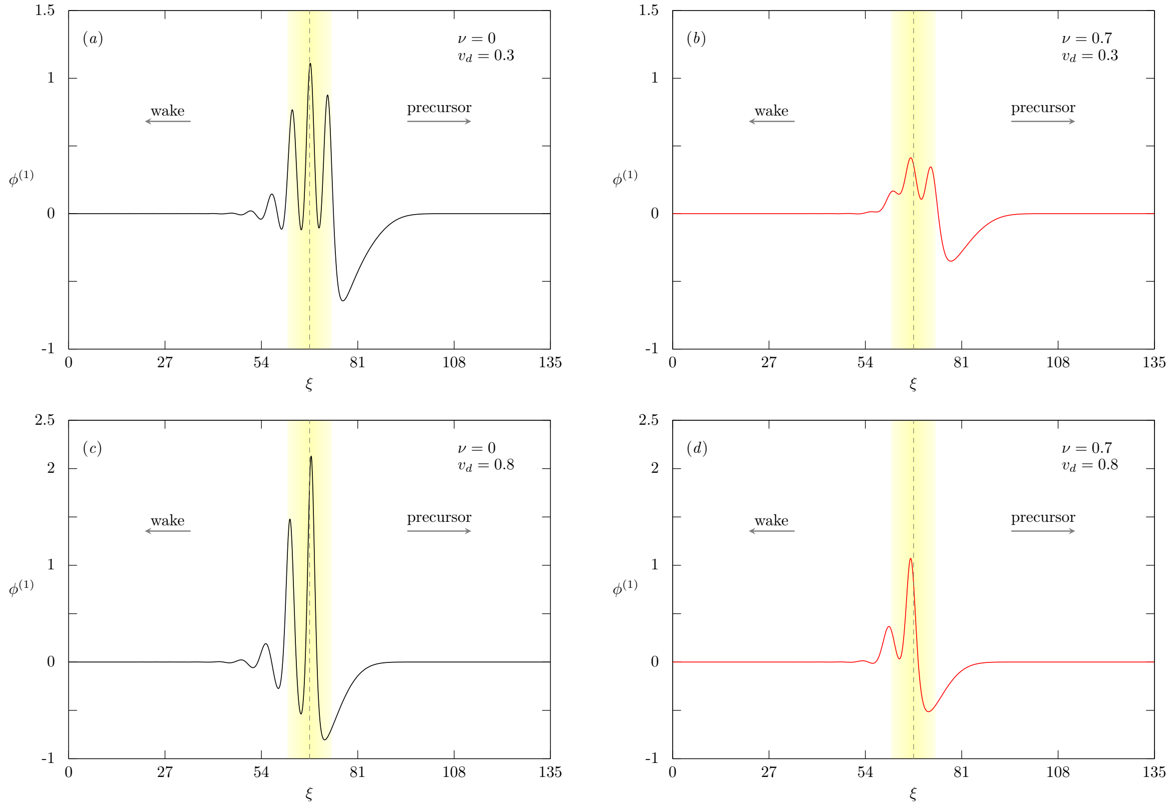

Figure 1: Solutions of the fKdV equation for negatively charged debris, where ν is the fluctuation frequency and v_d is the debris velocity normalized by the ion-acoustic speed. The figures are shown in the debris frame, indicated by the shaded vertical strip.

debris, which is computationally extensive and might not be consistent with the standard orbit motion limited (OML) theory. Besides, our externally controllable debris charge fluctuation can be seamlessly integrated into both the fKdV and PIC simulation setups. Additionally, the debris charge is assumed to be either positive definite or negative definite, while oscillating in time.

3 Forced KdV Dynamics

In order to derive the forced Korteweg-de Vries (fKdV) equation, we define the followingstretched coordinates

$$\xi = \epsilon^{1/2}(x - \lambda t), \quad \tau = \epsilon^{3/2} t, \tag{11}$$

where ϵ denotes a smallness parameter, which is $\ll 1$ and λ denotes the phase velocity of the ion-acoustic wave.

Following the reductive perturbation technique and considering a static and neutral equilibrium, we expand the variables n_i, v_i, and ϕ in powers of the parameter ϵ as

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{12}$$

$$v_i = \epsilon v_i^{(1)} + \epsilon^2 v_i^{(2)} + \cdots, \tag{13}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{14}$$

$$\rho_{ext} = \epsilon^2 \rho_{ext}^{(2)}, \tag{15}$$

where the external debris term has been considered as a second order perturbation [15]. As per the standard approach, at the lowest power of ϵ, we get the following relations for the first order perturbed quantities in terms

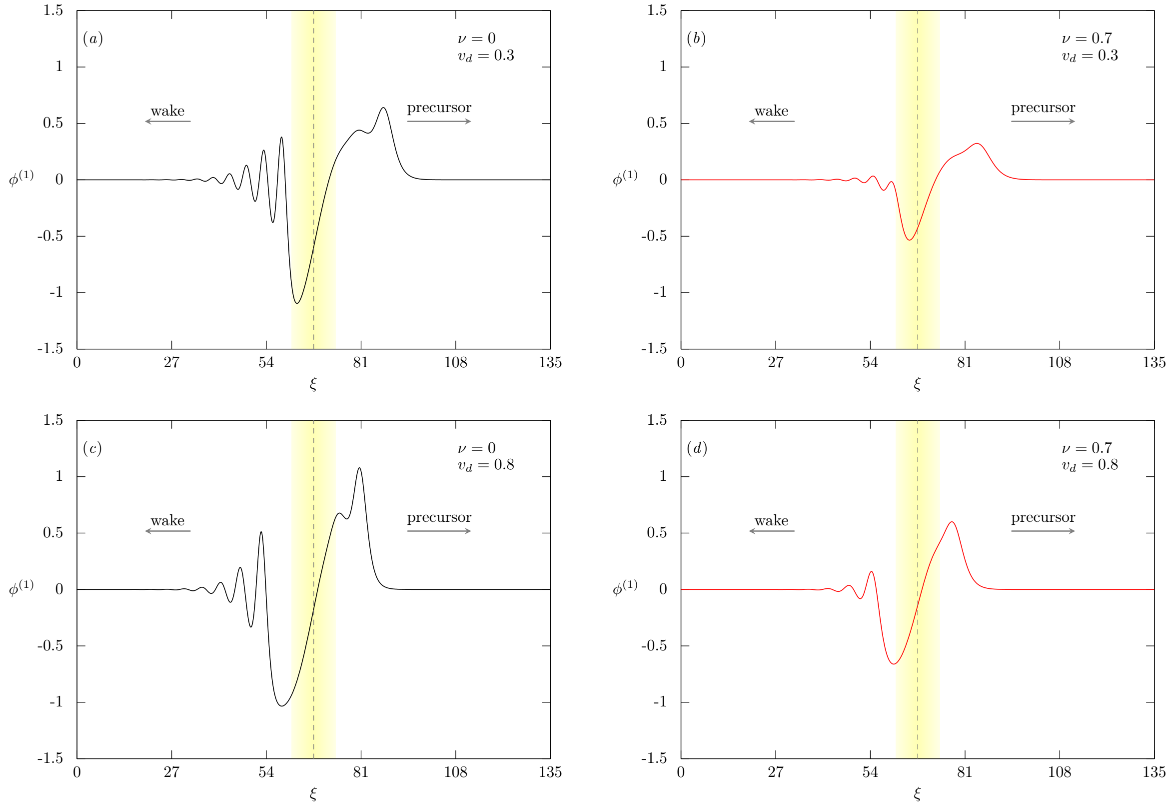

Figure 2: Solutions of the fKdV equation for positively charged debris, where ν is the fluctuation frequency and v_d is the debris velocity normalized by the ion-acoustic speed. The figures are shown in the debris frame, indicated by the shaded vertical strip.

of $\phi^{(1)}$

$$n_i^{(1)} = \frac{\phi^{(1)}}{\lambda^2 - 2\sigma}, \tag{16}$$

$$v_i^{(1)} = \frac{\lambda\phi^{(1)}}{\lambda^2 - 2\sigma}, \tag{17}$$

with the phase velocity (or the first compatibility condition) given by

$$\lambda = \sqrt{1 + 2\sigma}, \tag{18}$$

At the next higher power of ϵ, we eliminate the second order perturbed quantities using the first order relations (Eqs. 16 and 17), to get the fKdV equation as follows

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = B\frac{\partial\rho_{ext}^{(2)}}{\partial\xi}, \tag{19}$$

with A and B denoting the coefficients of nonlinearity and dispersion respectively, and are given as

$$A = \frac{1 + 3\sigma}{\sqrt{1 + 2\sigma}}, \tag{20}$$

$$B = \frac{1}{2\sqrt{1 + 2\sigma}}, \tag{21}$$

In Eq. 19, the debris term can be expressed as

$$\rho_{ext}^{(2)} = \rho_0 \exp\left(-\frac{(\xi - v_d\tau)^2}{2\delta^2}\right)\left(\frac{1 + \lfloor sin(\nu\tau)\rfloor}{2}\right), \tag{22}$$

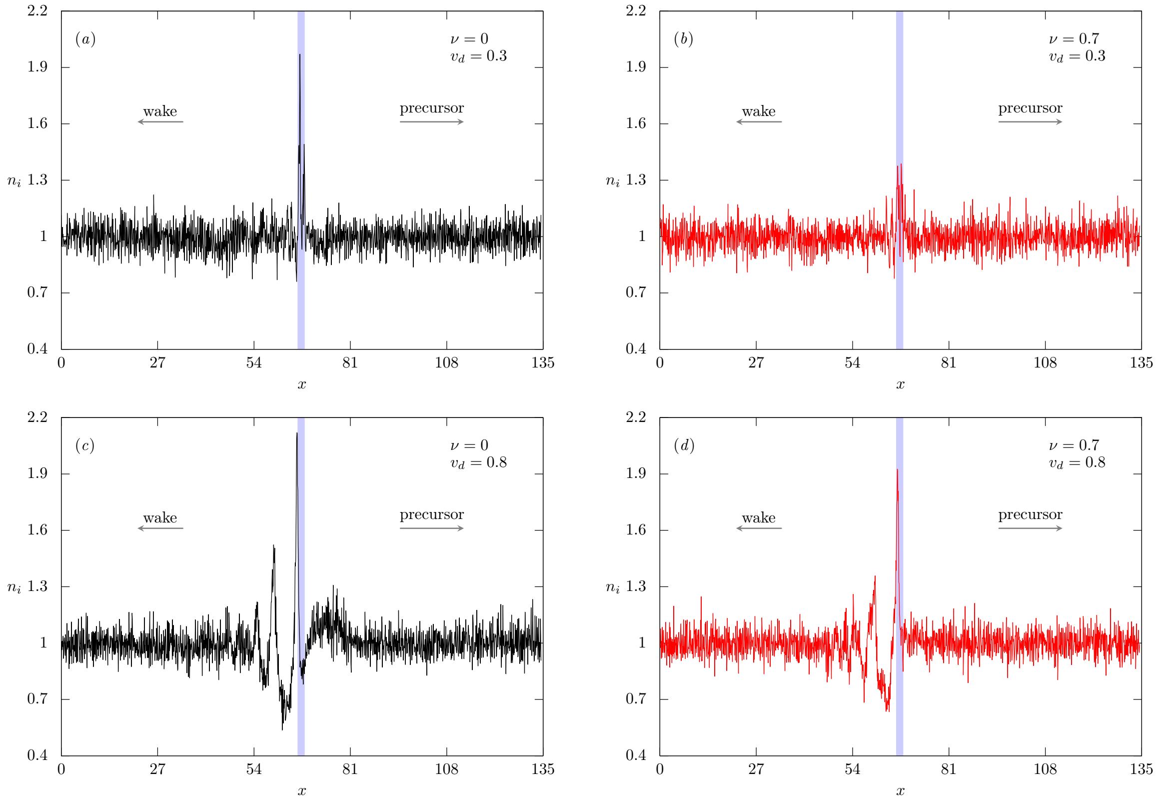

Figure 3: Ion density plots for negatively charged debris obtained from the PIC simulation after 20 ion plasma periods. The figures are shown in the debris frame, denoted by the vertical blue strip. The parameters ν and v_d are same as in Figs. 1 and 2.

where ρ_0 is the peak of the Gaussian profile of width δ moving with velocity v_d, normalized by the ion-sound speed. The term $(1 + \lfloor\sin(\nu\tau)\rfloor)/2$ modulates the debris charge by periodically turning it on and off at frequency ν.

The fKdV equation is numerically solved using the following parameters – $\rho_0 = 1.0$, $\sigma = 0.01$ and $\delta = 4.5$. The boundary conditions are assumed to be periodic with $\phi = 0$ being the initial condition. The numerical solutions for both negative and positively charged debris are now plotted in the rest frame of the debris for different values of v_d and ν in Figs. 1 and 2. For a negatively charged debris, the fKdV solutions exhibit pinned solitons localized at the debris site. These pinned solitons correspond to the ions trapped in the ion phase-space vortices, arising from the ion-ion counter streaming instability (IICSI) induced by the negatively charged debris. As the debris velocity increases, the pinned solitons begin to separate away from each other as the vortex widens, before ultimately disappearing. However, on applying a periodic fluctuation of the debris amplitude, the pinned solitons get suppressed and their formation appears to be delayed in Fig .1. When the positively charged, the plasma response is fundamentally different. In Fig .2, the positively charged debris creates a steepened precursor which evolves into a dispersive shock wave (DSW) along with an oscillatory wake. The DSW appears to be stable as the debris velocity increases. On applying the periodic fluctuation, a similar suppression of the DSW can be seen here as well.

4 PIC Simulation

In this section, we present the results obtained from a Particle-in-Cell (PIC) simulation of our e-i plasma model with the external charged debris under periodic charge fluctuation. The simulation has been performed using the h-PIC-MCC code [17, 18] which has been used in various electrostatic plasma studies like debris induced ion-ion counter streaming instability, dust-charge fluctuations, external debris induced chaotic dynamics, etc.

For relevance to the LEO region, the e-i number density of the plasma is taken as $n_{e,i} \sim 10^{16}\,\mathrm{m}^{-3}$ with

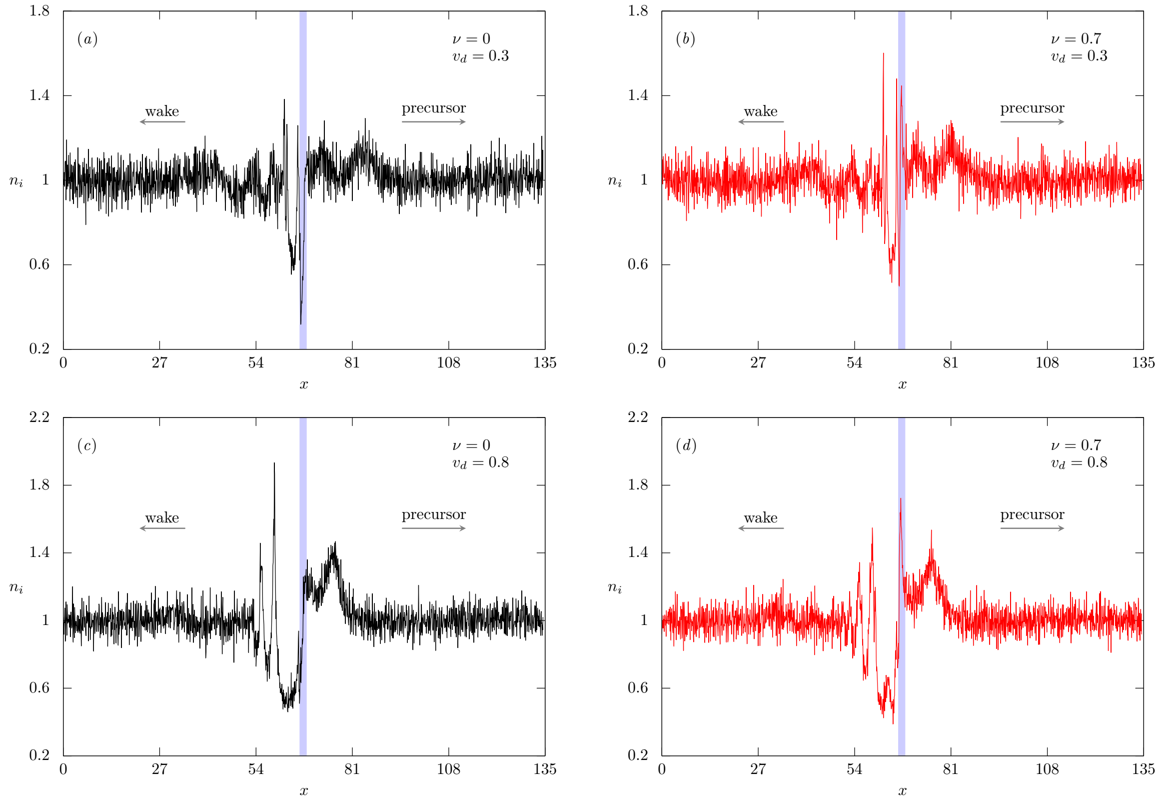

Figure 4: Ion density plots for positively charged debris obtained from the PIC simulation after 20 ion plasma periods. The figures are shown in the debris frame, indicated by the vertical blue strip. The parameters ν and v_d are same as in Figs. 1 and 2.

electron temperature $1\,\mathrm{eV}$ and ion temperature $0.01\,\mathrm{eV}$. The simulation domain has a length of $135\lambda_D$ equally discretized into 1600 cells, each having width $\sim 0.05\lambda_D$, fine enough to ensure proper resolution of Debye-scale structures. The external charged debris has been initialized at the centre of the simulation domain having width $\sim 2\lambda_D$. The simulation has been performed for 20 ion plasma periods, long enough to accurately capture the nonlinear structures induced by the debris. We should also note that the debris charge density must exceed a certain threshold value for the debris-induced nonlinear structures to emerge, otherwise the debris will be suppressed by the inherent particle noise, causing its effects to diminish rapidly. This threshold value for our plasma model happens to be 0.08 times the background charge density [5].

The simulation results for negatively charged debris are consistent with the fKdV results. However, the DSW excited by positively charged debris does not seem to be affected by the periodic debris charge fluctuation, which is contradicting the fKdV results. This discrepancy can be attributed to the fact that DSWs usually comprise multi-wavelength oscillations and their energy is spread over different wavenumbers. So, any damping in a particular wavenumber will not show up in a PIC simulation as it naturally evolves all the wavenumbers. In contrast to this, an fKdV solution will pick up only a single mode and it will show the damping (suppression) effectively.

5 Summary and Conclusion

In the present study, we have explored the various nonlinear structures generated by periodically fluctuating external charged debris in a flowing e-i plasma. Using the fluid equations of the plasma, we derived the forced Korteweg-de Vries (fKdV) equation, whose numerical solutions revealed the generation of pinned solitons for negatively charged debris and dispersive shock waves (DSWs) for positively charged debris. A key finding of the fKdV analysis was that a periodic fluctuation of the debris charge tends to suppress or delay the onset of these nonlinear structures.

To complement the fKdV results, a PIC simulation of the plasma was carried out with parameters relevant to the low Earth orbit (LEO) plasma. The simulation confirmed the suppression of pinned solitons for negatively charged debris. However, unlike the fKdV model, the DSWs excited by the positively charged debris were not significantly influenced by the periodic debris charge fluctuation. This discrepancy arises due to the PIC simulation inherently evolving multiple wavenumbers whereas the fKdV model captures only a single mode, making the suppression more apparent in the latter.

Our findings provide key insights into the effect of periodic charge fluctuation of space debris on the formation and evolution of nonlinear structures excited by the debris. These results are particularly relevant for understanding debris-plasma interactions in the LEO region, where monitoring such structures is key to the detection of space debris.

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